INDE5000 Supply Chain Modelling and Optimisation

Assignment, Semester One, [redacted]

[redacted]

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Question 1.

A wholesale distributor of bicycles is having trouble with shortages of a popular model (a small, one-speed girl's bicycle) and is currently reviewing the inventory policy for this model. The distributor purchases this model bicycle from the manufacturer monthly and then supplies it to various bicycle shops in the western United States in response to purchase orders. The monthly demand of the bicycles is 100,000 bikes. The distributor wants to use the EOQ model to determine his optimal inventory policy for bicycles and has determined that the followings are important:

- **The *ordering cost*, i.e., the cost of placing an order plus the cost of the bicycles being purchased, has two components: The administrative cost involved in placing an order is estimated as \$200, and the actual cost of each bicycle is \$35 for this wholesaler.**
- **The *holding cost,* i.e., the cost of maintaining an inventory, is \$1 per bicycle remaining at the end of the month. This cost represents the costs of capital tied up, warehouse space, insurance, taxes, and so on.**
- **The *shortage cost* is the cost of not having a bicycle on hand when needed. This particular model is easily reordered from the manufacturer, and stores usually accept a delay in delivery. Still, although shortages are permissible, the distributor feels that she incurs a loss, which she estimates to be \$15 per bicycle per month of shortage.**

Solution

(a) Determine the optimal production quantity each time, the cycle length if shortages are not permitted.

No shortage

a.) $Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2(100{,}000)(200+35Q)}{1}} = 7{,}000{,}005.71$

b.) $T = \frac{Q^*}{D} = \frac{7{,}000{,}005.71}{100{,}000} = 70$

(b) If planned shortages are permitted, find the maximum shortage, the cycle length and the optimal production quantity.

With shortage

a.) $Q^* = \sqrt{\frac{2DC_0}{C_h}\left(\frac{C_s+C_h}{C_s}\right)} = \sqrt{\frac{2(100{,}000)(200+35Q)}{1}\left(\frac{15+1}{15}\right)} = 7{,}466{,}672.38$

b.) $T = \frac{Q^*}{D} = \frac{7{,}466{,}672.38}{100{,}000} = 74.67$

c.) $Maximum\ shortage\ (S^*) = Q^*\left(\frac{C_h}{C_s+C_h}\right) = 7{,}466{,}672.38\left(\frac{1}{15+1}\right) = 466{,}667.02$

d.) $Opt. shortage\ level = Q^* - S^* = 7{,}466{,}672.38 - 466{,}667.02 = 7{,}000{,}005.36$

Question 2.

A television manufacturing company produces its own speakers, which are used in the production of its television sets. The television sets are assembled on a continuous production line at a rate of 8,000 per month, with one speaker needed per set. The speakers are produced in batches because they do not warrant setting up a continuous production line, and relatively large quantities can be produced in a short time. Therefore, the speakers are placed into inventory until they are needed for assembly into television sets on the production line. The company is interested in determining when to produce a batch of speakers and how many speakers to produce in each batch. Several costs must be considered:

- **Each time a batch is produced, a setup cost of $12,000 is incurred. This cost includes the cost of "tooling up," administrative costs, record keeping, and so forth. Note that the existence of this cost argues for producing speakers in large batches.**
- **The unit production cost of a single speaker (excluding the setup cost) is $10, independent of the batch size produced. (In general, however, the unit production cost need not be constant and may decrease with batch size.)**
- **The production of speakers in large batches leads to a large inventory. The estimated holding cost of keeping a speaker in stock is $0.30 per month. This cost includes the cost of capital tied up in inventory. Since the money invested in inventory cannot be used in other productive ways, this cost of capital consists of the lost return (referred to as the *opportunity cost*) because alternative uses of the money must be forgone. Other components of the holding cost include the cost of leasing the storage space, the cost of insurance against loss of inventory by fire, theft, or vandalism, taxes based on the value of the inventory, and the cost of personnel who oversee and protect the inventory.**
- **Company policy prohibits deliberately planning for shortages of any of its components. However, a shortage of speakers occasionally crops up, and it has been estimated that each speaker that is not available when required costs $1.10 per month.**

Solution

a) **Determine the optimal production quantity each time, the cycle length if shortages are not permitted.**

<u>*No shortage*</u>

a.) $Q^* = \sqrt{\frac{2(8000)(12000)}{0.3}} = 25298.22$

b.) $T = \frac{25298}{8000} = 3.16$

b) **If planned shortages are permitted, find the maximum shortage, the cycle length and the optimal production quantity.**

<u>*With shortage*</u>

a.) $Q^* = \sqrt{\frac{2(8000)(12000)}{0.3}\left(\frac{1.1+0.3}{1.1}\right)} = 28540.24$

b.) $T = \frac{28540}{8000} = 3.6$

c.) $Maximum\ shortage\ (S^*) = Q^*\left(\frac{C_h}{C_s+C_h}\right) = 28540.24\left(\frac{0.3}{1.1+0.3}\right) = 6115.77$

d.) $Opt.shortage\ level = Q^* - S^* = 28540.24 - 6115.77 = 22424.47$

Question 3.

(a) Prefab, a furniture manufacturer, uses 20,000 square feet of plywood per month. Their trucking company charges Prefab $400 per shipment, independent of the quantity purchased. The manufacturer offers an all unit quantity discount with a price of $1 per square foot for orders under 20,000 square feet, $0.98 per square foot for orders between 20,000 square feet and 40,000 square feet, and $0.96 per square foot for orders larger than 40,000 square feet. Prefab incurs a holding cost of 20 percent.

Solution

		Price	C_h	q_i	Q	Q adjust	TC
1	less than 20000	$1.00	0.2	0	30984	19999	246800.14
2	20000 - 40000	$0.98	0.196	20000	31298	31298	241334.50
3	more than 40000	$0.96	0.192	40001	31623	40001	236640.04

- **What is the optimal lot size for Prefab?**
  Optimal lot size is 40001 square feet per lot and obtain the discount price of $0.96 per square feet.
- **What is the annual cost of such a policy?**
  Annual cost is $236640.04
- **What is the cycle inventory of plywood at Prefab?**
  Cycle inventory = 40001/2 = 20000.5 units
- **How does it compare with the cycle inventory if the manufacturer does not offer a quantity discount but sells all plywood at $0.96 per square foot?**
  The optimal quantity will be 31623 units and the cycle inventory will e 1 811 5 u its Therefore, if the Prefab can order at the optimal quantity or as per the r req ir ment it will reduce the cycle inventory that result in reduce the holding cost.

(b) Reconsider Question 3(a) about Prefab. However, the manufacturer now offers marginal unit quantity discount for the plywood. The first 20,000 square feet of any order is sold at $1 per square foot, the next 20,000 square feet is sold at $0.98 per square foot, and any quantity over 40,000 square feet is sold for $0.96 per square foot.

Solution

		Price	C_h	q_i	R_i	Q	TC
1	first 20000	$1.00	0.2	0	0	30984	246800
2	20000 - 40000	$0.98	0.196	20000	20000	44263	243960
3	over 40000	$0.96	0.192	40001	39601	63246	242663.2

- **What is the optimal lot size for Prefab given this pricing structure?**

Optimal lot size is 63246 square feet per lot and obtain the discount price of $0.96 per square feet.

- **How much cycle inventory of plywood will Prefab carry given the ordering policy?**
  Cycle inventory = 63246/2 = 31623 units

Question 4.

Harvey Norway sells 3 sizes of beds: King size, Queen size, and Single size. Annual demands for the three-size products are D_K = 1,000 for the king-size beds, D_Q = 500 units for the queen-size beds, and D_S = 100 units for the single-size bed, and each model costs Harvey $1000, $800 and $400, respectively. A fixed transportation cost of $500 is incurred each time an order is delivered. For each model ordered and delivered on the same truck, an additional fixed cost of $150 is incurred for receiving and storage. Harvey Norway incurs a holding cost of 20 percent. Evaluate optimal lot sizes, order frequency, and annual inventory-related cost associated to each of the following.

- **Lots for each product are ordered and delivered independently.**

	King size	Queen size	Single size
Demand per year	1000	500	100
Each model cost	1000	800	400
Fixed transportation cost	500	500	500
Fixed cost of storage	150	150	150
Fixed ordering cost	650	650	650
Optimal order size	80.62	63.74	40.31
Cycle inventory	40.31	31.87	20.16
Annual holding cost	8062	5099.2	1612.8
Order frequency	12.4	7.84	2.48
Annual ordering cost	8062	5099.2	1612.8
Average flow time	0.040	0.063	0.202
Annual cost	16124	10198.4	3225.6

Therefore, annual inventory related cost is 29548

- **All three models are included each time an order is placed.**

$C_o^* = C_o + C_o^K + C_o^Q + C_o^S = 500 + 150 + 150 + 150 = 950$

Order frequency = $\sqrt{\frac{1000*(0.2)(1000)+500*(0.2)(800)+100*(0.2)(400)}{2(950)}} = 12.31$

	King size	Queen size	Single size
Demand per year	1000	500	100
Each model cost	1000	800	400
Fixed transportation cost	500	500	500
Fixed cost of storage	450	450	450
Fixed ordering cost	950	950	950
Optimal order size	81.23	40.62	8.12
Cycle inventory	40.62	20.31	4.06
Annual holding cost	8123.48	3249.39	324.94
Order frequency	12.31	12.31	12.31
Annual ordering cost	3898.17	3898.17	3898.17
Average flow time	0.041	0.041	0.041

Therefore, annual inventory related cost is 23392.32

- **Some products are ordered jointly.**

Step 1:

$$\overline{n_K} = \sqrt{\frac{IC_K D_K}{2(C_0 + C_0^K)}} = \sqrt{\frac{(0.2)(1000)(1000)}{2(500+150)}} = 12.40$$

$$\overline{n_Q} = \sqrt{\frac{IC_Q D_Q}{2(C_0 + C_0^Q)}} = \sqrt{\frac{(0.2)(800)(500)}{2(500+150)}} = 7.84$$

$$\overline{n_S} = \sqrt{\frac{IC_S D_S}{2(C_0 + C_0^S)}} = \sqrt{\frac{(0.2)(400)(100)}{2(500+150)}} = 2.48$$

Therefore, $\bar{n} = 12.40$

Step 2:

$$\overline{\overline{n_Q}} = \sqrt{\frac{IC_Q D_Q}{2C_0^Q}} = \sqrt{\frac{(0.2)(800)(500)}{2(150)}} = 16.32$$

$$\overline{\overline{n_S}} = \sqrt{\frac{IC_S D_S}{2C_0^S}} = \sqrt{\frac{(0.2)(400)(100)}{2(150)}} = 5.16$$

$$\overline{m}_Q = \frac{\bar{n}}{\bar{\bar{n}}_Q} = \frac{12.4}{16.32} = 0.76$$

$$\overline{m}_S = \frac{\bar{n}}{\bar{\bar{n}}_S} = \frac{12.4}{5.16} = 2.4$$

Therefore, $\overline{m}_Q = [0.76] = 1, \overline{m}_S = [2.4] = 3$

Step 3:

$$N = \sqrt{\frac{\sum IC_i m_i D_i}{2(C_0 + \sum C_0^i/m_i)}} = \sqrt{\frac{(0.2)(1000)(1000)+(0.2)(800)(1)(500)+(0.2)(400)(3)(100)}{2(500+150+\frac{150}{1}+\frac{150}{3})}} = 13.37$$

$n_K = 13.37$, $n_Q = 13.37$, $n_S = 4.46$

Step 4:

	King size	**Queen size**	**Single size**
Demand per year	1000	500	100
Each model cost	1000	800	400
Optimal order size	74.79	37.40	22.42
Cycle inventory	37.40	18.70	11.21
Annual holding cost	7480	2992	896.8
Average flow time	0.037	0.037	0.112

Annual ordering cost = $nC_o + n_K C_o^K + n_Q C_o^Q + n_S C_o^S$ = 11365

Therefore, annual inventory related cost is 22733.8

Verifying the results using EXCEL or MATLAB

Excel template or MATLAB code

Question 1

Given Data			
Demand	100000		
Order cost	200		
Unit Cost	35		
Holding cost	1		
shortage cost	15		
Q*	=Q*-SQRT(2*B4*(B5+(B6*Q*))/B7)=0		
#Quadratic Eq			
	Q^2	Q	C
	A	B	C
	1	-7000000	-40000000
X1	7000005.714		
X2	-5.71428105		
No shortage			
Q*	7000005.714		
T	70.00005714		
With shortage			
#Quadratic Eq			
	Q^2	Q	C
	A	B	C
	1	-7466666.667	-42666666.67
X1	7466672.381		
X2	-5.714281341		
Q*	7466672.381		
T	74.66672381		
Maximum shortage	466667.0238		
Opt. shortage level	7000005.357		

Question 2

Solution A				**Solution B (with shortage)**	
Given Data					
Order cost	$ 12,000.00			Order cost	$ 12,000.00
Demand	8000			Demand	8000
Holding cost	$ 0.30			Holding cost	$ 0.30
				Shortage cost	$ 1.10
Q*	25298.2213	<<a		Q*(with shortage)	28540.2427
T	3.16227766	<<b		T	3.56753034
				Maximum shortage(S*)	6115.7663
				Opt. shortage level	22424.4764

Question 3a

Discount Number	Discount Quantity	Discount Cost	
1	less than 20000	$1.00	
2	20000 to 40000	$0.98	
3	more than 40000	$0.96	
Input Data			
Demand	240000		
Ordering cost	400		
Carying cost (%)	20%		
	Range 1	Range 2	Range 3
Minimum quantity	0	20000	40001
Unit purchase cost	$1.00	$0.98	$0.96
Results			
	Range 1	Range 2	Range 3
EOQ (Q*)	30983.86677	31298.43186	31622.7766
Adjusted order quantity	19999	31298.43186	40001
Total holding cost	1999.9	3067.246322	3840.096
Total ordering cost	4800.240012	3067.246322	2399.940001
Total purchase cost	240000	235200	230400
Total cost	246800.14	241334.4926	236640.036
Optimal Lot size	40001		
Annual cost of such policy	236640.036		
Cycle inventory	20000.5		
If manufacturer does not offer a quantity discount but sells all plywood at $0.96 per square foot			
Optimal lot size	31622.7766		
Cycle inventory	15811.3883		

Question3b

Discount Number	Discount Quantity	Discount Cost
1	first 20000	$1.00
2	20000 to 40000	$0.98
3	more than 40000	$0.96

Input Data	
Demand	240000
Ordering cost	400
Carying cost (%)	20%

→ $Q_j^* = \sqrt{\frac{2(R_j - C_j q_j + C_o)D}{IC_j}}$

	Range 1	Range 2	Range 3
Minimum quantity	0	20000	40001
Unit purchase cost	$1.00	$0.98	$0.96
R	0	20000	39600.98

Results	Range 1	Range 2	Range 3
EOQ (Q*)	30983.86677	44262.66681	63245.94849
R-RCq+C0		800	1600.02
Total holding cost	2000	3960.098	6191.517055
Total ordering cost	4800	4799.880003	6071.611055
Total purchase cost	240000	235200	230400
Total cost	246800	243959.978	242663.1281
Optimal lot size	63245.94849		
Cycle inventory	31622.97424		

Question 4

Lots for each product are ordered and delivered independently.

		King size	Queen size	Single size
Demand per year	D	1,000	500	100
Product order cost	Co*	650	650	650
Unit cost	C	1000	800	400
Optimal order size	Q*	80.6226	63.7377	40.3113
Cycle length (year)	Q*/D	0.0806	0.1275	0.4031
Annual holding cost	(Q*/2)IC	8062.2577	5099.0195	1612.4515
Order frequency (n*)	(D/Q*)	12.4035	7.8446	2.4807
Annual ordering cost	(D/Q*)Co*	8062.2577	5099.0195	1612.4515
Average flow time	Q*/(2D)	0.0403	0.0637	0.2016
Inventory-related cost (TC)		16124.5155	10198.0390	3224.9031
Cycle Inventory	Q*/2	40.3113	31.8689	20.1556
Annual inventory	$ 29,547.46			

All three models are included each time an order is placed.

A holding cost of 20 percent	I	0.2	Fixed Trans cost	500		
Ordering cost	Co	$500.00				
	Co*	$950.00				
		King	Queen	Single		
Demand per year	D	1,000	500	100		
Product order cost	Co(i)	150	150	150		
Unit cost	C	1000	800	400		
	(D) (IC)	200000	80000	8000	12.3117	= n*
Optimal order size	Q*=D/n*	81.2233	40.6116	8.1223		
Cycle length	L=Q*/D	0.0812	0.0812	0.0812		
Annual holding cost	IHC=(Q*/2)(IC)	8122.3286	3248.9314	324.8931	11696.1532	= IHC
Average flow time	(Q*/2D)	0.0406	0.0406	0.0406		
Annual ordering cost $C_o^* n^*$					11696.1532	= OC
annual inventory-related cost					$ 23,392.31	= TC
cycle inventory	Q*/2	40.6116	20.3058	4.0612		

Some products are ordered jointly.

Step 1		King	Queen	Single	
N bar each size	N	12.40347346	7.844645406	2.480694692	
N bar	n	12.40347346			
Step 2			Queen	Single	
N bar bar			16.32993162	5.163977795	
M bar			0.759554525	2.401922307	
M bar(round up)			1	3	
Step 3					Sum
#calculation	C*m*D	1000000	400000	120000	1520000
#calculation	C0/m	150	150	50	350
N		13.37249152			
N each size		13.37249152	13.37249152	4.457497173	
Step 4					
		King	Queen	Single	**Total**
Demand Per year	D	1000	500	100	
Each model		1000	800	400	
Optimal order size		74.7803802	37.3901901	22.43411406	
Cycle inventory		37.3901901	18.69509505	11.21705703	
Annual holding cost	IHC=(Q*/2)(IC)	7478.03802	2991.215208	897.3645624	**11366.6178**
Average flow time		0.03739019	0.03739019	0.11217057	
	N	nk	nq	ns	
Annual Ordering cost	6686.245759	2005.873728	2005.873728	668.6245759	
Annual Ordering cost	11366.61779				
Total annual cost	Oc+Hc	22733.23558			